HYDRAULIC TURBOMACHINES
Exercises 2 - Velocity Triangles

Parametric Study for a Velocity Triangle of a Francis Turbine

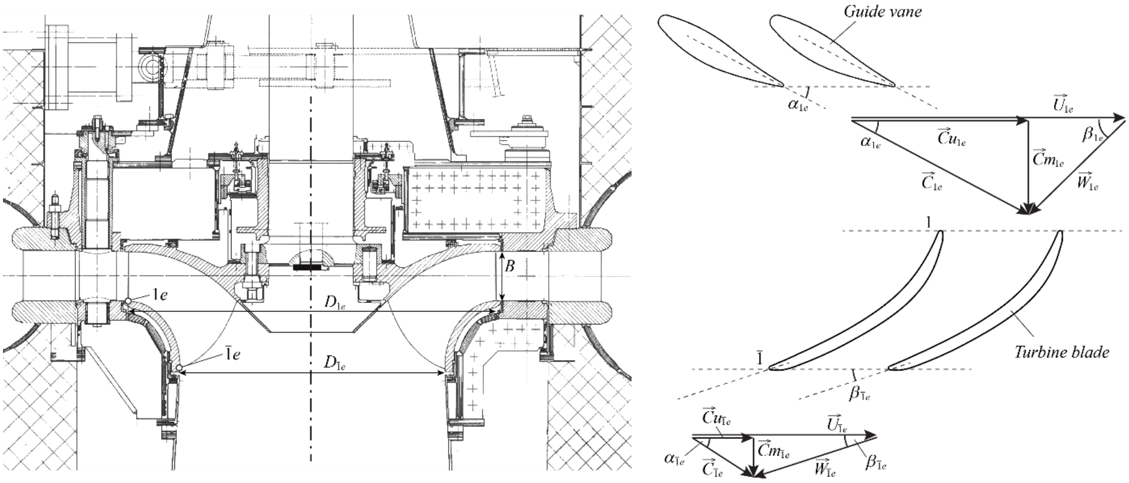


Figure 1. Scheme of the hydropower plant with its characteristics.

The meridional view of a Francis turbine and one example of the velocity triangle are sketched in Figure 1. For a Francis turbine, the angle of the absolute flow velocity at the inlet α_{1e} corresponds to the guide vane opening degree, and the angle of the relative flow velocity at the outlet corresponds to the outlet blade angle $\beta_{\bar{1}e}$, as shown in Figure 1. Referring to the figure, answer the following questions.

1) Give the expression of turbine rotational velocity U_{1e} and $U_{\bar{1}e}$ as functions of the angular rotation ω and the inlet and outlet diameters, D_{1e} and $D_{\bar{1}e}$ respectively.

$$U_{1e} = \frac{D_{1e}\omega}{2}$$

$$U_{\bar{1}e} = \frac{D_{\bar{1}e}\omega}{2}$$

2) Give the relation of the turbine discharge Q and the discharge Q_t traversing the runner as a function of the volumetric efficiency η_v.

$$Q_t = \eta_v Q$$

3) Give the meridional components of the flow velocity Cm_{1e} and $Cm_{\bar{1}e}$ as functions of the discharge Q and the volumetric efficiency η_v by using the geometrical parameters defined in the left part of Figure 1.

$$Cm_{1e} = \frac{Q_t}{A_{1e}} = \frac{\eta_v Q}{\pi D_{1e} B},$$

$$Cm_{\bar{1}e} = \frac{Q_t}{A_{\bar{1}e}} = \frac{4\eta_v Q}{\pi D_{\bar{1}e}^2}$$

4) Considering the vectorial relationship at the turbine runner inlet 1, write the relation of π, Cu_{1e}, Q, η_v, D_{1e}, B and α_{1e}.

$$Cu_{1e} = \frac{1}{\tan\alpha_{1e}} Cm_{1e} = \frac{1}{\tan\alpha_{1e}} \frac{\eta_v Q}{\pi D_{1e} B}$$

5) Considering the vectorial relationship at the turbine runner outlet $\bar{1}$, derive the relation of π, $Cu_{\bar{1}e}$, $U_{\bar{1}e}$, Q, η_v, $D_{\bar{1}e}$ and $\beta_{\bar{1}e}$.

Using the trigonometric relationship $\tan\beta_{\bar{1}e} = \dfrac{Cm_{\bar{1}e}}{U_{\bar{1}e} - Cu_{\bar{1}e}}$ *and knowing* $Cm_{\bar{1}e}$ *from question 3):*

$$Cu_{\bar{1}e} = U_{\bar{1}e} - \frac{1}{\tan\beta_{\bar{1}e}} \frac{4\eta_v Q}{\pi D_{\bar{1}e}^2}$$

6) Derive the relation of the transformed specific energy E_t as a function of U_{1e}, $U_{\bar{1}e}$, Q, η_v, D_{1e}, $D_{\bar{1}e}$, B, α_{1e} and $\beta_{\bar{1}e}$.

Using the relationship from Lecture 3 and the equations found in questions 4) and 5) for Cu_{1e} *and* $Cu_{\bar{1}e}$*:*

$$E_t = Cu_{1e} U_{1e} - \frac{1}{2} Cu_{\bar{1}e} U_{\bar{1}e} = \frac{1}{\tan\alpha_{1e}} \frac{\eta_v Q}{\pi D_{1e} B} U_{1e} - \frac{1}{2}\left(U_{\bar{1}e} - \frac{1}{\tan\beta_{\bar{1}e}} \frac{4\eta_v Q}{\pi D_{\bar{1}e}^2} \right) U_{\bar{1}e}$$

7) Considering the ratio of $\dfrac{U_{1e}}{U_{\bar{1}e}}$, derive the relation for transformed specific energy E_t as a function of $U_{\bar{1}e}$, Q, η_v, $D_{\bar{1}e}$, B, α_{1e} and $\beta_{\bar{1}e}$.

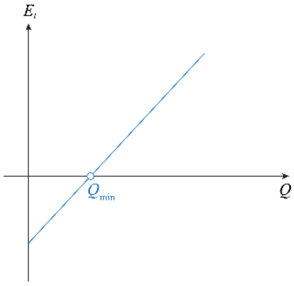
Considering the ratio $\dfrac{U_{1e}}{U_{\bar{1}e}} = \dfrac{D_{1e}}{D_{\bar{1}e}}$*, the transformed energy* E_t *can be written as:*

$$\begin{aligned}
E_t &= \frac{1}{\tan\alpha_{1e}} \frac{\eta_v Q}{\pi D_{1e} B} U_{1e} - \frac{1}{2}\left(U_{\bar{1}e} - \frac{1}{\tan\beta_{\bar{1}e}} \frac{4\eta_v Q}{\pi D_{\bar{1}e}^2} \right) U_{\bar{1}e} \\
&= \frac{1}{\tan\alpha_{1e}} \frac{\eta_v Q}{\pi D_{1e} B} U_{\bar{1}e} \frac{D_{1e}}{D_{\bar{1}e}} - \frac{1}{2}\left(U_{\bar{1}e} - \frac{1}{\tan\beta_{\bar{1}e}} \frac{4\eta_v Q}{\pi D_{\bar{1}e}^2} \right) U_{\bar{1}e} \\
&= -\frac{1}{2} U_{\bar{1}e}^2 + \left(\frac{1}{\tan\alpha_{1e}} \frac{D_{\bar{1}e}}{B} + \frac{2}{\tan\beta_{\bar{1}e}} \right) \frac{\eta_v Q}{\pi D_{\bar{1}e}^2} U_{\bar{1}e}
\end{aligned}$$

8) For a given rotational frequency of the runner, sketch the transformed specific energy E_t as a function of the traversing discharge Q_t, and derive the condition of minimum discharge Q_t^{min} to achieve positive specific energy.

For a given rotational frequency, the transformed power E_t *is linearly increased as a function of* $Q_t = \eta_v Q$*:*

$$E_t = -\frac{1}{2}U_{\bar{1}e}^2 + \left(\frac{1}{\tan\alpha_{1e}}\frac{D_{\bar{1}e}}{B} + \frac{2}{\tan\beta_{\bar{1}e}}\right)\frac{Q_t}{\pi D_{\bar{1}e}^2}U_{\bar{1}e}$$



In order to produce energy ($E_t > 0$), Q_t must be greater than:

$$Q_{t\_min} = \frac{\pi D_{\bar{1}e}^2 U_{\bar{1}e}}{2\left(\frac{1}{\tan\alpha_{1e}}\frac{D_{\bar{1}e}}{B} + \frac{2}{\tan\beta_{\bar{1}e}}\right)}$$

9) When the turbine is operated at the best efficiency point (BEP), express the transformed power P_t with necessary variables, considering the assumption of the best efficiency point $\left(Cu_{\bar{1}e} = 0\right)$.

$$E_{t\_BEP} = \frac{1}{\tan\alpha_{1e}}\frac{\eta_v Q}{\pi D_{1e} B}U_{1e}, \qquad P_{t\_BEP} = \rho\eta_v Q E_t = \rho\frac{1}{\tan\alpha_{1e}}\frac{\eta_v^2 Q^2}{\pi D_{1e} B}U_{1e}$$

Calculation of the best efficiency using a hill-chart

The $Q_{ED} - n_{ED}$ hill-chart of a Francis turbine with the iso-value curves of both the global efficiency η (red curves) and guide vane opening α (blue curves) is represented in Figure 3. The horizontal and vertical axes represent IEC discharge factor Q_{ED} and IEC speed factor n_{ED}, respectively. Using the hill-chart, answer the following questions. Use the following values if required, where η_v and η_{me} are the volumetric and mechanical efficiency, respectively:

$$D_{1e} = 4.20 \text{ m},\ D_{\bar{1}e} = 3.50 \text{ m},\ B = 0.60 \text{ m},\ n = 3.88 \text{ Hz},\ \eta_v = 0.98 \text{ and } \eta_{me} = 0.97$$

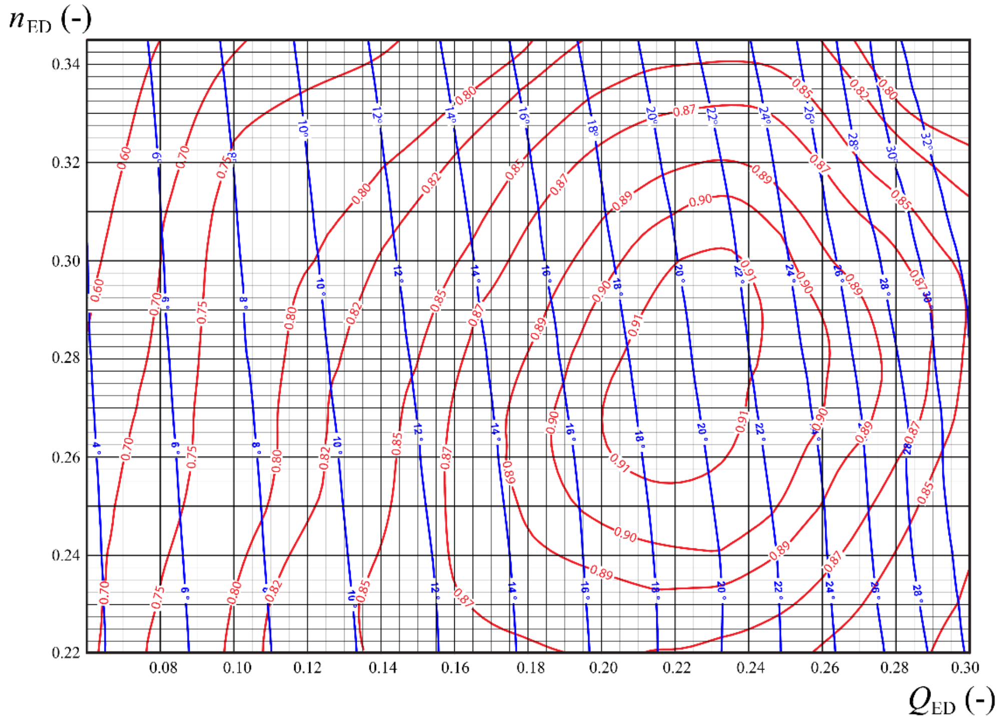


Figure 2. $Q_{ED} - n_{ED}$ *hill-chart of a Francis turbine*

10) Point out the best efficiency point in the hill-chart, and estimate the global efficiency $\eta_{BEP}^{hill-chart}$ and guide vane opening $\alpha_{BEP}^{hill-chart}$ at the best efficiency point (BEP).

The BEP is located at approximately at the center of the isoline with highest efficiency. Therefore, we can approximately expect an efficiency of 92-93% at the BEP by doing a qualitatively estimation on the hillchart in figure 2.

For this machine, the estimated best efficiency is $\eta_{BEP}^{estimated} \cong 0.925$*, and is achieved for 20° guide vane opening angle,* $Q_{ED} = 0.225$ *and* $n_{ED} = 0.28$*.*

11) At the BEP, the available head H and the discharge in the power plant Q are measured as $H = 235$ m and $Q = 130$ m^3 s^{-1}. Calculate the transformed energy E_t at the best efficiency point. Then, calculate the available power at the BEP, i.e. P^{BEP}.

From question 9) we know that $E_{t,BEP} = \dfrac{1}{\tan\alpha_{1e}} \dfrac{\eta_v Q}{\pi D_{1e} B} U_{1e}$

Using $\omega = 2\pi n = 24.38\ rad/s$ *and* $U_{1e} = \dfrac{D_{1e}\omega}{2} = 51.20\ m/s$ *(see question 1)), we can compute the transformed energy by considering a guide vane opening angle of 20 degrees as found in question 10):*

$E_t^{BEP} = 2263.72\ \mathrm{J\,kg^{-1}}$.

Therefore, the available power (or output power) is:

$P_{BEP} = \eta_{me} P_t = \rho \eta_{me} \eta_v Q E_t = 279.75\,\mathrm{MW}$